

# Effects of long-range Coulomb interaction on the quantum transport in fractional quantum Hall edges

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We study the effects of long-range Coulomb interaction (LRCI) on the quantum transport in FQH edges with  $\nu = 1/(2k + 1)$ . We consider two models, i.e., the quasi-particle tunneling (QPT) model and the electron tunneling (ET) model at the point contact. The tunneling conductance  $G(T)$  is obtained using the renormalization group treatment. In QPT model, it is found that LRCI further reduces  $G(T)$  below a crossover temperature  $\Lambda_w$ . In ET model, on the other hand, there is a temperature region where LRCI enhances  $G(T)$ , and nonmonotonic temperature dependence is predicted.

Keywords: electron-electron interactions, Tomonaga-Luttinger liquid, fractional quantum Hall effect

Quantum Hall liquid is an incompressible liquid with the gap in the charge excitation [1]. Then the low-lying excitations are localized near the edge of the sample, which determine the low energy physics of the incompressible liquid. These edge modes of a fractional quantum Hall (FQH) system are considered to be described as a chiral Tomonaga-Luttinger (TL) liquid [2, 3], and recent experiments seem to support this idea showing the power law dependence of the conductance on the temperature and voltage [4, 5]. Consider a two-terminal Hall bar geometry where the bulk FQH liquid has both upper and lower edges. We assume that the bulk system exhibits the FQH effect with a filling factor  $\nu = 1/(2k + 1)$ . In this case it is expected that only one edge mode exists for each of the edge when the confining potential is steep. The edge modes for upper and lower edges have the opposite chiralities. By applying the negative gate voltage one can introduce the depleted region of electrons squeezing the Hall bar. This structure, called point contact, introduces interaction between the upper and lower edges, i.e., the backward scattering between the edges due to the quasiparticle tunneling (QPT) through the bulk FQH liquid. This can be described by a TL model with a scattering potential at  $x = 0$  (QPT model) [6, 7]. This model predicts a low temperature tunneling conductance as  $G(T) \sim T^{2/\nu-2}$  [6, 7, 8, 9, 10], which is consistent with the recent experiment [4]. Another model is the electron tunneling (ET) model, where the depleted region is considered to be a vacuum, and the electron can tunnel through this region between the left and right FQH liquids. This model also predicts  $G(T) \sim T^{2/\nu-2}$ . Recently, however, Moon and Girvin (MG) pointed out a discrepancy between the above theory and the experiments at very low temperatures [11]. They propose that this discrepancy is resolved by incorporating the effects of long-range Coulomb interaction (LRCI) in the ET model [11].

In this paper, we study the effects of LRCI on

the quantum transport [12, 13, 14] in the above two models of FQH edges. We obtain the tunneling conductance  $G(T)$  through the potential barrier using the renormalization group treatment, and show that QPT and ET models give qualitatively different behaviors for the low temperature conductance, which depend on the length scale of the system. In the following we employ the unit where  $\hbar = k_B = 1$ .

*Quasiparticle tunneling model* - The model describes a two-terminal Hall bar geometry where a two-dimensional electron system between the left and right terminals has upper and lower edges with a scattering potential at  $x = 0$ .

$$S = S_0 + S_a + S_w + u \int d\tau \cos \phi_+(\tau, x = 0) \quad (1)$$

The first term  $S_0$  describes the usual chiral TL liquid [2, 3, 15],

$$\mathcal{L}_0 = \frac{v_R}{8\pi\nu} \left\{ \left( \frac{\partial \phi_+}{\partial x} \right)^2 + \left( \frac{\partial \phi_-}{\partial x} \right)^2 \right\} + \frac{i}{4\pi\nu} \frac{\partial \phi_+}{\partial \tau} \frac{\partial \phi_-}{\partial x}$$

where the short-range interactions are included in the velocity  $v_R$ . The second and third terms correspond to the intra- and inter-edge Coulomb interactions respectively [11],

$$S_a = \frac{1}{2} \int dx dy V_a(x - y) \{ \rho_u(x) \rho_u(y) + \rho_l(x) \rho_l(y) \}$$

$$S_w = \int dx dy V_w(x - y) \rho_u(x) \rho_l(y),$$

where

$$V_a(x) = \frac{e^2}{\epsilon \sqrt{x^2 + a^2}}, \quad V_w(x) = \frac{e^2}{\epsilon \sqrt{x^2 + w^2}}$$

with  $a$  being an ultraviolet cutoff on the scale of lattice constant and  $\epsilon$  being the dielectric constant. Bosonization of the densities on upper and lower edges are given by

$$\rho_{u,l}(x) = \frac{1}{2\pi} \frac{\partial}{\partial x} \phi_{u,l}(x)$$

with  $\phi_{\pm} = \phi_u \pm \phi_l$ . The last term in Eq.(1) describes a QPT through the bulk FQH liquid  $\psi_u^\dagger \psi_l + \psi_l^\dagger \psi_u$ , since the creation and annihilation of quasiparticles on the upper and lower edges are described respectively by the operators  $\psi_{u,l} = e^{\pm i\phi_{u,l}}$ . Integrating out  $\phi_-$ , we obtain an effective action for  $\phi_+$ ,

$$= \frac{1}{\beta} \sum_{\omega} \int \frac{dk}{2\pi} \frac{v_R}{8\pi\nu} \left\{ \eta_+ k^2 + \frac{\omega^2}{\eta_- v_R^2} \right\} |\phi_+|^2, \quad (2)$$

where  $\eta_{\pm}(k) = 1 + \frac{\nu}{2\pi} \{V_a(k) \pm V_w(k)\}$  with  $V_a(k)$  and  $V_w(k)$  being the fourier transformation of  $V_a(x)$  and  $V_w(x)$ . Noting that  $a \ll w$ , we can evaluate  $\eta_{\pm}(k)$  as

$$\begin{aligned} \eta_+(k) &\sim 1 + \xi \ln \frac{1}{|k|a}, \\ \eta_-(k) &= \begin{cases} 1 + \xi \ln \frac{w}{a} & (|k|w \ll 1) \\ 1 + \xi \ln \frac{1}{|k|a} & (|k|w \gg 1) \end{cases} \end{aligned}$$

where  $\xi = (\nu\alpha/\pi\epsilon)(c/v)$  measures the strength of the inter-edge Coulomb interaction, with  $\alpha = e^2/\hbar c$  being the fine structure constant. Eq. (2) gives the dispersion relation  $\omega(k) = v_R |k| \sqrt{\eta_+(k)\eta_-(k)}$  [16, 17]. The problem of the tunneling through a single barrier in TL liquids was first studied by Kane and Fisher[6] and later extended by Furusaki and Nagaosa[7]. They derived an effective action for the phase field at the barrier site by integrating out the continuum degrees of freedom.

$$S_{QPT}[\theta] = \frac{1}{4\pi\beta\nu} \sum_{\omega} \zeta_w(\omega) |\omega| |\theta(\omega)|^2 + u \int d\tau \cos \theta(\tau),$$

where  $\theta$  is the phase at the point contact, and

$$\zeta_w(\omega) \sim \sqrt{\frac{\eta_+(k_0)}{\eta_-(k_0)}} = \begin{cases} \sqrt{1 + 2\xi \ln \frac{v_R}{\omega w}} & (\omega < \Lambda_w) \\ 1 & (\omega > \Lambda_w) \end{cases}$$

with  $\Lambda_w = v_R/w$  and  $k_0 = |\omega|/\sqrt{\eta_+(k_0)\eta_-(k_0)}$ .

First we discuss the RG analysis at high temperatures, we consider the limit where the scattering potential is very weak. We study the scaling behavior of the scattering potential using the standard perturbative RG treatment [6, 7]. The scaling equation for  $u$  is derived perturbatively by successively integrating over the high frequency components, and the result is [6, 7, 18],

$$\frac{d(u/\Lambda)}{u/\Lambda} = \left( \frac{\nu}{\zeta_w(\Lambda)} - 1 \right) \frac{d\Lambda}{\Lambda}. \quad (3)$$

It turns out that as the cut-off  $\Lambda$ , which can be replaced by the temperature  $T$ , the scattering potential  $u$  scales to stronger values, i.e., relevant. The above

perturbative treatment with respect to  $u$  breaks down as  $u/\Lambda$  becomes the order of unity, where the crossover from weak to strong coupling occurs. This crossover temperature  $\Lambda_1$  is obtained by setting  $u(\Lambda_1)/\Lambda_1 = 1$  and is given by  $\Lambda_1 = (u_0/\Lambda_0^\nu)^{1/(1-\nu)}$ , where  $u_0$  and  $\Lambda_0$  are the bare strength of the potential and cutoff, respectively. Then the consideration here is restricted to the higher temperatures  $T > \Lambda_1$ . When  $\Lambda_1 < \Lambda_w$ ,  $u(\Lambda)$  exhibits two different behaviors corresponding to the two temperature regions. Above  $\Lambda_w$ , LRCI has no effect on the RG equation, and  $u(\Lambda) \propto \Lambda^{\nu-1}$ . The temperature dependence of  $G(T)$  is obtained by the second order perturbation in the renormalized coupling constant  $u(\Lambda)$  with the cut-off  $\Lambda$  being replaced by the temperature  $T$ . Then we obtain the usual temperature dependence  $G(T) - \nu e^2/h \sim -T^{2\nu-2}$ . When  $\Lambda_1 < \Lambda_w$ , there is the temperature region  $\Lambda_1 < T < \Lambda_w$ , where the solution of Eq.(3) is given by

$$u(\Lambda) = u(\Lambda_w) \exp \left[ -\frac{\nu}{\xi} \left\{ \sqrt{1 + 2\xi \ln \frac{\Lambda_w}{\Lambda}} - 1 \right\} \right],$$

leading to the tunneling conductance

$$\begin{aligned} G(T) - \nu \frac{e^2}{h} \\ \sim -\frac{1}{T^2} \exp \left[ -\frac{2\nu}{\xi} \left\{ \sqrt{1 + 2\xi \ln \frac{\Lambda_w}{T}} - 1 \right\} \right]. \end{aligned} \quad (4)$$

If we expand the exponent in Eq. (4), the leading correction to  $G(T) - \nu \frac{e^2}{h}$  is

$$-\left( \frac{T}{\Lambda_w} \right)^{2\nu-2} \exp \left[ 2\nu\xi \left( \ln \frac{\Lambda_w}{T} \right)^2 \right],$$

which means that the LRCI further reduces  $G(T)$ .

Next we discuss the RG analysis at low temperatures, where we consider the opposite limit where the scattering potential is very strong. This corresponds to the low temperature  $T < \Lambda_1$ . In this limit, the electron transport can be viewed as the tunneling of the phase  $\theta$  from a potential minimum to an adjacent minimum. This process corresponds to an instanton or an anti-instanton. By the duality mapping in the dilute instanton gas approximation (DIGA), we transform the original model to an analogous model in the weak potential limit [19],

$$S_{DIGA}[\tilde{\theta}] = \frac{\nu}{4\pi\beta} \sum_{\omega} \frac{|\omega|}{\zeta_w(\omega)} |\tilde{\theta}(\omega)|^2 + 2z \int d\tau \cos \tilde{\theta}(\tau),$$

where  $z$  is the instanton fugacity, which is the tunneling matrix elements from  $\theta = 0$  to  $\pm 2\pi$ . It turns

out that only when  $\xi = 0$ , which corresponds to the case of short range interactions, the dual action can be identified with the original one in terms of the correspondences  $\nu \leftrightarrow 1/\nu$ ,  $\tilde{\theta} \leftrightarrow \theta$  and  $2z \leftrightarrow u$ . According to the standard perturbative RG treatment for the instanton fugacity  $z$ , we obtain

$$\frac{d(z/\Lambda)}{z/\Lambda} = \left( \frac{\zeta_w(\Lambda)}{\nu} - 1 \right) \frac{d\Lambda}{\Lambda}.$$

$z(\Lambda)$  exhibits a crossover with a characteristic temperature  $\Lambda_w$  when  $\Lambda_w < \Lambda_1$ . In the region  $\Lambda_w < T < \Lambda_1$ , the LRCI has no effect on the RG equation. When  $T < \Lambda_w$ , the above RG equation can be solved in the same way as the high temperature case. We find that the tunneling of an instanton is suppressed by the LRCI. The conductance  $G(T)$  is obtained in the second order perturbation in the renormalized fugacity  $z(\Lambda)$ , i.e., the tunneling amplitude, with the cut-off  $\Lambda$  being replaced by  $T$ :

$$G \sim \frac{1}{T^2} \exp \left[ -\frac{2}{3\nu\xi} \left\{ \left( 1 + 2\xi \ln \frac{\Lambda_w}{T} \right)^{3/2} - 1 \right\} \right],$$

which has the asymptotic forms for  $\Lambda_2 \ll T < \Lambda_w$ ,

$$G \sim T^{2/\nu-2} \exp \left[ -\frac{\xi}{\nu} \left( \ln \frac{\Lambda_w}{T} \right)^2 \right],$$

while for  $T \ll \Lambda_2$ ,

$$G \sim \frac{1}{T^2} \exp \left[ -\frac{4}{3\nu} \sqrt{2\xi} \left( \ln \frac{\Lambda_w}{T} \right)^{3/2} \right],$$

where  $\Lambda_2 = \Lambda_w e^{-1/2\xi}$ . At very low temperatures, tunneling conductance decreases faster than any power law. Here we evaluate the crossover temperatures. We assume that bulk FQH liquid exhibits the  $\nu = 1/3$  plateau. According to MG, we use  $v_R \sim 10^5 \text{ m/s}$ ,  $\xi \sim 0.2$ , and  $w \sim 60 \mu\text{m}$ . In the QPT model these values give  $\Lambda_w \sim 10 \text{ mK}$ , and  $\Lambda_2$  becomes the order of  $1 \text{ mK}$ .  $\Lambda_1$  is controlled by the strength of the potential barrier at the point contact. Then we believe that the effect of LRCI in the QPT model might resolve one of the discrepancies between the theoretical prediction and the experiment pointed out by MG.[11]

*Electron tunneling model-* Here we consider the model where the bulk FQH liquid is divided into left and right condensates with a characteristic separation  $d \ll w$ , and electrons can tunnel between the left and right edges through an insulating region at the point contact (Fig. 2).

We start with the case where  $w$  is sufficiently large compared to the energy scales in question. In this

case left and right edges are assumed to be parallel and infinitely long. Experimentally such a situation can be realized by putting a thin film insulator (with width  $d \ll w$ ) between the FQH liquids. Using this experimental geometry, the rise of the tunneling conductance will be observed experimentally. We start with the effective action for the phase at the point contact  $\theta$ :

$$S_{ET}[\theta] = \frac{1}{4\pi\beta\nu} \sum_{\omega} \zeta_d(\omega) |\omega| |\theta(\omega)|^2 + \gamma \int d\tau \cos \frac{\theta(\tau)}{\nu}, \quad (5)$$

where  $\zeta_d(\Lambda)$  has the same form as  $\zeta_w(\Lambda)$  with  $\Lambda_w$  replaced by  $\Lambda_d = v_R/d$ . Here, note that the phase  $\theta$  in the ET model has a mathematically equivalent but physically different origin from the one in the QPT model, due to the different origins of two chiral TL liquids. The first term of Eq. (5) describes the edge modes of the left and right FQH liquid with intra- and inter-edge Coulomb interactions. The second term comes from ET between the left and right edges:  $\Psi_L^\dagger \Psi_R + \Psi_R^\dagger \Psi_L \sim \cos \frac{\phi_{\pm}}{\nu}$ , where the electron operators on the left and right edges are given by  $\Psi_{L,R} \sim \exp[\pm i\phi_{L,R}/\nu]$  with  $\phi_{\pm} = \phi_L \pm \phi_R$ , and  $\gamma$  is the strength of ET. In Eq. (5), we have already integrated out the continuum degrees of freedom. Following the standard perturbative RG treatment, we obtain

$$\frac{d(\gamma/\Lambda)}{\gamma/\Lambda} = \left( \frac{1}{\nu\zeta_d(\Lambda)} - 1 \right) \frac{d\Lambda}{\Lambda}.$$

As is the previous cases, above the crossover temperature  $\Lambda_d$ , LRCI has no effect on the RG equation, i.e.  $G(T) \sim T^{2/\nu-2}$ . Below the crossover temperature  $\Lambda_d$ , it is easy to see that LRCI makes  $G(T)$  decrease more gradually than  $T^{2/\nu-2}$ . What is more drastic is, however, that the ET becomes relevant when the temperature is further lowered below  $\Lambda_3 = \Lambda_d \exp[-(1/\nu^2 - 1)/2\xi]$ . As the temperature is lowered from  $\Lambda_d$ ,  $G(T)$  decreases more gradually than  $T^{2/\nu-2}$ , and at  $\Lambda_3$  it turns to the increase if  $\Lambda_3 > \Lambda_w$ .

When the temperature is further lowered below  $\Lambda_w$ , one has to take care of the edges extended to right and left rather than the edges facing to each other (Fig. 2). To decide the present situation we start with the usual QPT model with LRCI. After incorporating the LRCI, we throw away the  $x > 0$  part and require the constraint that  $\Psi_L + \Psi_R = 0$  at  $x = 0$  for the QPT model, which means  $\phi_L(x, t) = -\phi_R(-x, t) + \nu\pi$  [20]. Thus we obtain the left branch of the ET model at  $T < \Lambda_w$ . Making the right branch in the same way, we study the tunneling between them to obtain the same low temperature dependence of  $G(T)$  as the QPT model.

Now we can explicitly write down our prediction for the tunneling conductance in the ET model. As the temperature is lowered from  $\Lambda_d \gg \Lambda_w$ ,  $G(T)$  decreases more gradually than  $T^{2/\nu-2}$ , and at  $\Lambda_3$  it turns to the increase. At lower temperatures than  $\Lambda_d$ ,  $G(T)$  scales as

$$G(T) \sim \frac{\Lambda_d}{T^2} \exp \left[ -\frac{2}{\nu\xi} \left( \sqrt{1 + 2\xi \ln \frac{\Lambda_d}{T}} - 1 \right) \right]$$

which have the following asymptotic form for  $T \gg \Lambda_3$ ,

$$G(T) \sim T^{2/\nu-2} \exp \left[ \frac{\xi}{\nu} \left( \ln \frac{\Lambda_d}{T} \right)^2 \right],$$

which means that the conductance is enhanced compared with the  $T^{2/\nu-2}$  for the short-range interaction case. In the region  $\Lambda_w \ll T \ll \Lambda_3$ ,  $G(T)$  scales as  $T^{-2}$ . At very low temperatures ( $T < \Lambda_w$ ), LRCI further reduces  $G(T)$  as in the QPT model.

Our treatment of ET model is not equivalent to the one in MG [11], where we believe that the charge phase and the Josephson phase are confused, although the final results are similar to ours. The Josephson phase is a phase of  $\Psi^{L\dagger} \Psi^{R\dagger}$ , and its gradient is proportional to the current and not to the density. ET should be described as a cosine potential in terms of the charge phase.

In summary, we study the effects of LRCI on the quantum transport in FQH edges with  $\nu = 1/(2k+1)$ . We consider two models, i.e., quasi-particle tunneling (QPT) model and electron tunneling (ET) model. Various crossovers of the tunneling conductance  $G(T)$  as a function of the temperature  $T$  are found. In the QPT model the LRCI reduces the conductance  $G(T)$  compared with the case of short range interaction. In the ET model, on the other hand, there is a temperature region where  $G(T)$  is enhanced, and even the nonmonotonic temperature dependence is possible.

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